Government General Degree College Chapra
Department of Mathematics
PO and CO (NEP)
Fourth Semester

Programme Outcome:

1. Acquire a comprehensive understanding of fundamental mathematical principles, including algebra, calculus, differential equations, vector spaces, and abstract structures such as groups, rings, and fields.

2. Apply logical reasoning and mathematical techniques to analyze and solve both theoretical and real-life problems involving pure and applied mathematics.

3. Demonstrate proficiency in mathematical modeling and problem-solving using differential equations, calculus and linear systems to address problems in science, engineering, and economics.

4. Utilize modern abstract structures such as groups, rings, fields, and vector spaces to explore symmetry, structure, and transformations, with applications in cryptography, coding theory, and automorphisms.

5. Interpret results through graphical and numerical approaches, using plotting and visualization techniques to study the behavior of solutions of differential equations.

6. Demonstrate the ability to communicate mathematical ideas clearly and effectively, both in written and verbal form, and construct sound logical proofs across diverse topics.

10. Foster a research-oriented mindset and prepare for higher studies in mathematics or related interdisciplinary fields by cultivating critical thinking and a strong theoretical base.

11. Appreciate the historical development and societal relevance of mathematics, understanding its evolution and utility in the digital age.

12. Uphold ethical practices and responsibility in the application of mathematics, ensuring accuracy, integrity, and respect for the broader impacts of mathematical work.

Course Outcome:

Course code & title	Syllabus	Course Outcome
MATH-M-T-04: Differential Equations	**Unit 1. [15L]** Differential equations and mathematical models. General, particular, explicit, implicit and singular solutions of a differential equation. Separable equations and equations reducible to this form. Exact differential equations and integrating factors. Linear equation and Bernoulli equations, special integrating factors and transformations. First order and higher degree differential equations, solvable for x, y and p, Clairaut's Equations: general and singular solutions. **Unit 2. [15L]** Lipschitz condition and Picard's Theorem (Statement only). General solution of homogeneous equation of second order, principle of superposition. Wronskian: its properties and applications, linear homogeneous and non-homogeneous equations of higher order with constant coefficients. Euler's equation, method of undetermined coefficients. Method of variation of parameters. **Unit 3. [15L]** Systems of linear differential equations. Types of linear systems., Differential operators. An operator method for linear systems with constant coefficients. Basic Theory of linear systems in normal form. Homogeneous linear systems with constant coefficients, two Equations in two unknown functions. **Unit 4. [10L]** Equilibrium points.	After successfully completing this course, the student will be able to: **CO1**: Understand and classify different types of differential equations (ordinary and partial), including general, particular, explicit, implicit, and singular solutions. **CO2**: Solve first-order differential equations using various standard methods such as separation of variables, integrating factors, and transformations (including linear and Bernoulli equations). **CO3**: Apply analytical techniques like Picard's theorem and Lipschitz condition (conceptually) to study existence and uniqueness of solutions. **CO4**: Solve second and higher-order linear differential equations using methods such as undetermined coefficients, variation of parameters, Euler's equations, and Wronskian-based analysis. **CO5**: Analyze and solve systems of linear differential equations using operator methods and normal form techniques, and understand the structure of solutions for homogeneous linear systems. **CO6**: Interpret equilibrium points and phase planes, and understand qualitative behavior of differential equations through graphical methods. **CO7**: Solve differential equations using power series about

	Interpretation of the phase plane. Power series solution of a differential equation about an ordinary point, solution about a regular singular point. **Unit 5. [15L]** Partial differential equations – Basic concepts and definitions. Mathematical problems. First order equations: classification, construction and geometrical interpretation, Lagrange's method, Charpit's method. Method of characteristics for obtaining general solution of quasi linear equations. Canonical forms of first-order linear equations. Method of separation of variables for solving first order partial differential equations. **Graphical demonstration (Teaching aid) [5L]** 1. Plotting a family of curves which are solutions of second order differential equations. 2. Plotting a family of curves which are solutions of third order differential equations.	ordinary and regular singular points. **CO8**: Understand the fundamentals of partial differential equations and solve.
MATH-M-T-05: Algebra-II	**Unit 1. [20L]** Properties of cosets. Lagrange's theorem and consequences including Fermat's little theorem. External direct product of a finite number of groups. Center of a group, centralizer, normalizer. Normal subgroups. Factor groups. Cauchy's theorem for finite abelian groups. Group homomorphisms, basic properties of homomorphisms. Cayley's theorem.	After successfully completing this course, the student will be able to: **CO1**: Understand and apply fundamental group theory concepts such as cosets, Lagrange's theorem, Fermat's little theorem, normal subgroups, factor groups, and group homomorphisms. **CO2**: Analyze structural properties of groups including center, centralizer, normalizer, automorphisms, characteristic subgroups, and commutator subgroups.

	Properties of isomorphisms. First, second and third isomorphism theorems. Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups. Characteristic subgroups, Commutator subgroups and its properties. **Unit 2. [10L]** Properties of external direct products, the group of units modulo n as an external direct product, internal direct products. Fundamental theorem of finite abelian groups. Sylow's theorems and consequences. Cauchy's theorem, Simplicity of A_n for $n \geq 5$, non-simplicity tests. **Unit 3. [15L]** Definition and examples of rings. Properties of rings, Subrings. Integral domains and fields. Characteristics of a ring. Ideal, ideal generated by a subset of a ring. Factor rings. Operations on ideals. Prime and maximal ideals. Ring homomorphisms, properties of ring homomorphisms. Isomorphism theorems I, II and III. **Unit 4: [15L]** Concept of Vector space over a field: Examples, concepts of Linear combinations, linear dependence and independence of a finite number of vectors. Sub- space, concepts of generators and basis of a finite dimensional vector space.	**CO3**: Demonstrate understanding of isomorphism theorems and apply them in group-theoretic contexts; verify Cayley's theorem and examine automorphism groups of cyclic groups. **CO4**: Explore internal and external direct products, and apply the fundamental theorem of finite abelian groups and Sylow's theorems to classify finite groups. **CO5**: Understand the definition and properties of rings, subrings, integral domains, fields, and identify different types of ideals (prime, maximal); apply isomorphism theorems in ring theory. **CO6**: Gain conceptual clarity in vector spaces, linear dependence and independence, subspaces, basis, dimension, and apply theorems like the replacement, extension, and deletion theorems. **CO7**: Understand row space, column space, and apply Gram-Schmidt orthogonalization in Euclidean spaces. **CO8**: Comprehend and work with linear transformations, their matrix representations, and analyze null space, range, rank, and nullity. **CO9**: Find eigenvalues, eigenvectors, and characteristic polynomials; apply the Cayley-Hamilton theorem and perform matrix diagonalization and canonical form transformations.

	Replacement theorem. Extension theorem. Deletion theorem and their applications. Row space, column space. Euclidean Spaces. Orthogonal and orthonormal vectors. Gram-Schmidt process of orthogonalization **Unit 5. [15L]** Linear transformations. Null space. Range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations. Eigenvalues, eigen vectors and characteristic equation of a matrix. Matric polynomials, Cayley-Hamilton theorem and its use in finding the inverse of a matrix. Diagonalization, Canonical forms.	
MATH-MI-T-02 Calculus & Differential Equations	Real-valued functions defined on an interval, limit and Continuity of a function (using $\varepsilon-$). Algebra of limits. Differentiability of a function. Successive derivative: Leibnitz's theorem and its application to problems of type e^{ax+b}sinx, $e^{ax+b}cosx$, $(ax+b)inx$, $(ax+b)^n cosx$. Partial derivatives. Euler's theorem on homogeneous function of two and three variables. Curvature, rectilinear asymptotes. Indeterminate Forms: L'Hospital's Rule (Statement and Problems only). Statement of Rolle's Theorem and its geometrical interpretation. Mean value theorems of Lagrange and Cauchy. Statements of Taylor's and Maclaurin's theorems with Lagrange's and Cauchy's forms of remainders. Taylor's and Maclaurin's infinite series of functions like e^x, sinx, cosx, $(1+x)^n$, log(1+x) with restrictions wherever necessary. Application of the principle of maxima and minima for a function of a single variable. **Unit 2. [5L]**	After completing this course, students will be able to: **CO1**: Understand the concept of limit, continuity, and differentiability of real-valued functions using the ε-δ definition. **CO2**: Compute successive derivatives using Leibnitz's theorem and solve related problems involving exponential and trigonometric functions. **CO3**: Evaluate partial derivatives and apply Euler's theorem for homogeneous functions in two and three variables. **CO4**: Apply L'Hospital's Rule to solve indeterminate forms and find curvature and rectilinear asymptotes. **CO5**: State and interpret Rolle's, Lagrange's, and Cauchy's Mean Value Theorems geometrically and algebraically.

	Reduction formulae, derivations and illustrations of reduction formulae of the type $\int sin_n x\, dx, \int cos_n x dx, \int tan_n x dx,$ $\int sec_n x dx, \int (\log x)_n dx,$ $\int sin_n x cos_m x dx.$ **Unit 3. [20L]** First order equations: (i) Exact equations and those reducible to such equations. (ii) Euler's and Bernoulli's equations (Linear). (iii) Clairaut's Equations: General and Singular solutions. Second order differential equation: (i) Method of variation of parameters, (ii) Method of undetermined coefficients. Linear homogeneous equations with constant coefficients, method of variation of parameters, simultaneous differential equations.	**CO6**: Use Taylor's and Maclaurin's Theorems (with remainder forms) to expand functions and analyze convergence. **CO7**: Apply differential calculus to identify maxima and minima of single-variable functions. **CO8**: Derive and apply reduction formulas for trigonometric and logarithmic integrals. **CO9**: Solve first-order and second-order differential equations using exact, Bernoulli's, Clairaut's methods, and the method of variation of parameters. **CO10**: Analyze linear differential equations with constant coefficients and apply methods of undetermined coefficients and simultaneous equations.